

Error Analysis of Multi User CDMA System over α - μ Fading Channel

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Abstract: In this paper error performance of multi-user code division multiple access (CDMA) system with block coding and interleaving over alpha mu (α - μ) fading channel is analysed. It is shown by analysis that with increase in number of users SNR decreases hence the error performance of the system deteriorates. This has been established by simulation over α - μ channel for various α and μ values.

Keywords: α - μ fading, random interleaving, bit error rate, code division multiple access, Walsh-Hadamard code, multi-user interference.

I. INTRODUCTION

Coding is applied to reduce the probability of error in wireless system. The error correction enhances the performance on the cost of decrease in data rate or increase in signal bandwidth. Code division multiple access (CDMA) is a multiple access technology. In multiuser systems there are multiple users who share the common radio channel. This mechanism is termed as 'Multiple Access' technology. CDMA overcomes the problems of frequency division multiple access, the first generation cellular system, and time division multiple access technology the second generation global systems for mobile (GSM) systems, by assigning a unique code sequence to each user pair to spread the information signal across the assigned frequency band. The information signal is retrieved at receiver by correlation of received signal with the same assigned code for that user pair. This multiple access system is called as CDMA or spread spectrum multiple access (SSMA). The 3G systems such as wideband WCDMA, high speed downlink/uplink packet access HSDPA/HSUPA, CDMA 2000 are based on code division, where different users are allocated different codes.

The performance of direct sequence code division multiple access (DS-CDMA) over multipath fading channel have been analysed in [7] and bit error rate (BER) performance of wireless communication system compared with multiple paths using Rake receiver for additive white Gaussian noise (AWGN), Rayleigh fading and Rician fading channel. The error performance of multiuser CDMA system with space-time coding is studied in [8] in Rician fading channel, and the corresponding BER and symbol error rate analysis are presented. Suppression of multiuser interference (MUI) and mitigation of multipath effects constitute major challenges in the design of third-generation wireless mobile systems. Most wideband and multicarrier uplink CDMA schemes suppress MUI statistically in the presence of unknown multipath. A mutually orthogonal user code receiver system have been

presented in [9] for quasi-synchronous blind CDMA that eliminates MUI deterministically and mitigates fading irrespective of the unknown multipath. Rotated Walsh-Hadamard spreading matrices have been investigated in [10] for a broadband multi-carrier MC-CDMA system with robust channel estimation in the synchronous downlink.

The paper is organized as follows. In Section 2, the α - μ fading model and its Probability density function is briefly discussed. In Section 3, the CDMA system performance is discussed. In Section 4, Walsh-Hadamard code generation technique is described. In Section 5, random interleaving method is explained. Monte-Carlo simulation results for BER performance of CDMA system over α - μ fading channel with interleaving in different α and μ condition are presented in Section 6. The paper is concluded by Section 7.

II. THE α - μ FADING MODEL

The multipath fading in wireless communication is modelled by several distribution such as Rayleigh, Rician, Weibull, Nakagami. In the recent past alpha-mu (α - μ) fading model [1] has been proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The α - μ represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading condition. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ . The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

Table-1: Algorithm for generation of α - μ distributed random variable

1.	Procedure α - μ random variable generation
2.	$\alpha \leftarrow$ Channel Parameter

3.	$\mu \leftarrow$ Channel Parameter
4.	$x \leftarrow$ Number of random variables
5.	$\Omega \leftarrow$ Mean
6.	$\sigma^2 \leftarrow$ Variance
7.	$H \leftarrow$ zero matrix of order $1 \times x$
8.	for $i \leftarrow 1$ to μ do
9.	$H = H +$ matrix of order $1 \times x$ having complex Gaussian random variable i.e. $X(\Omega, \sigma^2) + j X(\Omega, \sigma^2)$
10.	end for
11.	Fading envelope $\leftarrow H^\alpha$
12.	end procedure

In [1, 2] the α - μ fading distribution and its probability density function has been described. In the α - μ distribution, it is considered that a signal is composed of clusters of multipath waves. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components.

The α - μ probability density function (PDF), $f_R(r)$ of envelope R is given as

$$f_R(r) = \frac{\alpha \mu^\mu r^{\alpha\mu-1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right] \quad (1)$$

where $\alpha > 0$ is the power parameter, and α -root mean value of R^α is given as

$$\hat{r} = \alpha \sqrt{E(R^\alpha)} = \alpha \sqrt{2\mu\sigma^2}$$

where $\mu \geq 0$, is the inverse of the normalized variance of α - μ envelope R^α , and

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad \text{is the Gamma function.}$$

Algorithm given in Table.1 is used for generation of α - μ distributed random variable in the simulation work reported in this paper. Analytical and simulated results for PDF of fading envelope of α - μ fading channel defined by eq. (1), are shown in Fig.1. It is verified that both the analytical and simulated results are matching.

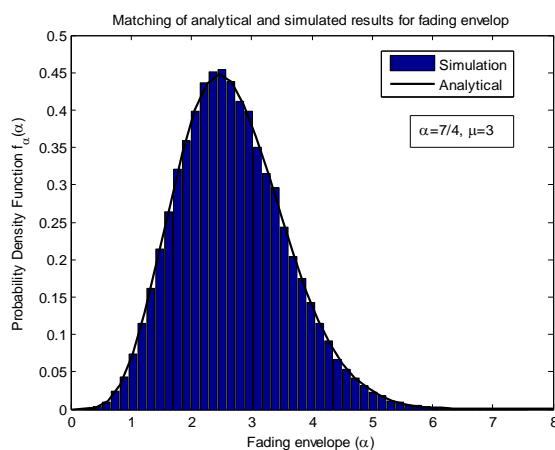


Fig.1. Matching of analytical and simulated results for α - μ fading envelope

III. CDMA SYSTEMS

CDMA system is also known as a “Spread Spectrum” system since the bandwidth or spectrum of the original signal is spread in this system. In CDMA, spectrum is spread by a factor of N , where N is code length for each symbol. The CDMA codes are known as “Spreading code”, and length of code N is called “Spreading factor”. Any two codes in CDMA systems are orthogonal codes. Thus, the number of orthogonal codes corresponding to length N is N . Hence, the number of users that can access the common channel is N . The spreading codes are also known as pseudo noise (PN) sequence, because they look like a random sequence of $+1$ and -1 , but actually they are not random. One technique to generate spreading codes of longer length is through, linear feedback shift register (LFSR).

Let in a multi-user CDMA, $C_i(n)$ be code sequence and a_i be symbol corresponding to i^{th} user.

$$\text{Transmitted CDMA Signal, } x(n) = \sum_{i=0}^{k-1} a_i C_i(n) \quad (2)$$

$$\text{Received Signal, } y(n) = h \times \left(\sum_{i=0}^{k-1} a_i C_i(n) \right) + w(n) \quad (3)$$

At receiver signal of each user is retrieved by correlation with code of the respective user. Accordingly for j^{th} user, correlated output will be;

$$r_j = \frac{1}{N} \sum_n y(n) \times C_j(n) \quad (4)$$

$$r_j = \frac{1}{N} \sum_n \left(h \times \left(\sum_{i=0}^{k-1} a_i C_i(n) \right) + w(n) \right) \times C_j(n) \quad (5)$$

The multiplication of C_i and C_j on right hand side will have one term $i = j$ for desired user, terms corresponding to $i \neq j$ are undesired users causing multi-user interference (MUI), and a noise term regarding $w(n)$. Calculating the power for all the terms.

The term $i = j$, for desired user

$$\begin{aligned} \text{Desired signal} &= \frac{1}{N} \sum_n h \times (a_j C_j(n)) C_j(n) \\ &= h a_j \frac{1}{N} \sum_n C_j^2(n) = h a_j \end{aligned} \quad (6)$$

$$\text{Since } C_j(n) \text{ is } +1 \text{ or } -1, \text{ hence } \frac{1}{N} \sum_n C_j^2(n) = \frac{1}{N} \sum_n 1 = \frac{N}{N} = 1$$

$$\text{Power of desired } j^{th} \text{ user} = |h|^2 |a_j|^2 = |h|^2 P_j \quad (7)$$

There are total k users, and i^{th} user may be from $i=0$ to $k-1$. Out of these k users, j^{th} user is desired user. Hence, there will be $k-1$ undesired users causing MUI for the desired user i.e. j^{th} user. Let us evaluate MUI caused by these $k-1$ undesired users to j^{th} desired user. For this, first we will evaluate MUI by one of the undesired user, say ($i=0$), out of $k-1$ undesired users.

Here mean of cross correlation, r_{0j} will be

$$E\{r_{0j}\} = E\left\{\frac{1}{N} \sum_n C_0(n)C_j(n)\right\} = \frac{1}{N} \sum_n E\{C_0(n)C_j(n)\} = 0 \quad (8)$$

And power of cross correlation, r_{0j} will be

$$\begin{aligned} E\{|r_{0j}|^2\} &= E\left\{\frac{1}{N} \sum_n C_0(n)C_j(n) \times \frac{1}{N} \sum_m C_0(m)C_j(m)\right\} \\ &= E\left\{\frac{1}{N^2} \sum_n \sum_m C_0(n)C_j(n)C_0(m)C_j(m)\right\} \\ &= \frac{1}{N^2} \sum_n \sum_m E\{C_0(n)C_0(m)\} \times E\{C_j(n)C_j(m)\} \\ &= \frac{1}{N^2} \sum_n E\{C_0^2(n)\} \times E\{C_j^2(n)\} = \frac{1}{N^2} \sum_n 1 \times 1 = \frac{N}{N^2} = \frac{1}{N} \quad (9) \end{aligned}$$

Since, terms of $m \neq n$ will be zero because codes are orthogonal. Now, the MUI by the undesired user ($i=0$), out of $k-1$ undesired users, for j^{th} desired user, may be given with help of eq.(5) and (9).

MUI by ($i=0$) undesired user

$$= E\{|h|^2 \times |a_0|^2 \times |r_{0j}|^2\} = |h|^2 P_0 \frac{1}{N} \quad (10)$$

Therefore MUI by all the $k-1$ undesired users will be

$$= |h|^2 \frac{1}{N} \sum_{i=0, i \neq j}^{k-1} P_i \quad (11)$$

Finally the Noise power, $\frac{1}{N^2} \sum_n E[\omega^2(n)]E[C_j^2(n)]$

$$= \frac{1}{N^2} \sigma_\omega^2 \sum_n 1 = \frac{N \sigma_\omega^2}{N^2} = \frac{\sigma_\omega^2}{N} \quad (12)$$

Therefore, signal to (interference +noise) ratio, SINR for j^{th} user will be

$$\text{SINR} (\gamma_j) = \frac{|h|^2 P_j}{|h|^2 \left[\sum_{i=0, i \neq j}^{k-1} P_i \right] + \sigma_\omega^2} \times N \quad (13)$$

IV. WALSH - HADAMARD CODE

Walsh-Hadamard code having length $N = T_s / T_c$ and synchronized in time, are orthogonal for a symbol, where T_s is symbol time and T_c is chip time. Thus the cross correlation of any two codes is zero. Since synchronizing of users is possible on downlink, because of all signals originating from same transmitter, but challenging in uplink case. Hence this code is mostly used for downlink channels.

Walsh-Hadamard sequence of length N are obtained from the rows of an $N \times N$ Hadamard matrix H_N . For $N=2$ the Hadamard matrix is given as eq. (14).

$$H_2 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \quad (14)$$

Larger Hadamard matrices with help of H_2 and so on can be obtained from eq.(15)

$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & -H_N \end{bmatrix} \quad (15)$$

Each row of H_N provides different chip sequence, thus the number of spreading code in Walsh-Hadamard code is N . The direct sequence spread spectrum (DSSS) with Walsh-Hadamard codes can support maximum $N = T_s / T_c$ users. Substituting $N=2$ in eq. (14), we get codes for 4 users, as shown in each row of eq.(16).

$$H_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \quad (16)$$

V. RANDOM INTERLEAVING

Unlike wireline AWGN, in wireless due to multipath and fading, channel may exhibits bursty error characteristic, that is signal fading due to multipath propagation often causes the signal to fall below the noise level, resulting in large number of errors i.e. burst errors. Such error clusters are not usually corrected by codes designed for statistically independent errors. Suppose a burst of errors of length b has occurred which means a sequence of b -bit in errors. Then, the burst error correction capability of a systematic (n, k) block code, which has $(n-k)$ parity check bits, is $b < \lceil \frac{1}{2}(n - k) \rceil$.

An effective method to improve performance of coding in fading channels for dealing with burst error channels is to interleave the coded data in such a way that the bursty channel is transformed to a channel having independent errors. Thus coding is typically combined with interleaving to mitigate the effect of error bursts.

The encoded data are processed by the interleaver and transmitted over the channel. At the receiver, the deinterleaver puts the data in proper sequence and passes them to the decoder. As a result of the interleaving/deinterleaving, error bursts are spread out in time so that errors within a code word appear to be independent. In a random interleaver, a block of N input bits are written in the interleaver in the same order in which they are received. Then, they are read out in a random manner. A random interleaver is a random permutation π . The interleaver has a corresponding deinterleaver (π^{-1}) that acts on the interleaved data sequence and restores it to the original order.

If the input data sequence is $U = [u_1, u_2, \dots, u_N]$, then permuted data sequence is $U \times P$, where P being the interleaving matrix with single 1, which is randomly located in each row and column, all other entries being zero. The de-interleaving matrix (P^T) is transpose of the interleaving matrix (P).

VI. SIMULATION RESULTS & DISCUSSIONS

BER performance for multi-user CDMA system over α - μ fading channel is obtained by Monte-Carlo simulation. The CDMA communication system shown in Fig.2 has been considered in this simulation. BER performance simulated results for different α and μ values are shown in Fig. 3 to Fig.6. In these simulation 100000 bits have been considered for a particular α and μ combination. The codeword $n = 7$, and each message is having binary sequence of length $k = 4$. The α - μ fading channel is assumed to be stationary for each codeword and changes independently for different codewords. The generator matrix given in eq. (17) is used by encoder for generating codewords.

$$G_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (17)$$

The Walsh-Hadamard code H_2, H_4, H_8 , and H_{16} for 2, 4, 8, and 16 users respectively is obtained from eq. (15). Energy requirement for different users data for transmission will increase, as we increase the value of N. Therefore, for faithful comparison we have normalized the user data from the value of N. In this paper we have used second order Walsh-Hadamard matrix for transmission of two user data, fourth order matrix for transmission of four user data and so on. This scheme will utilize minimum spectrum when users are less.

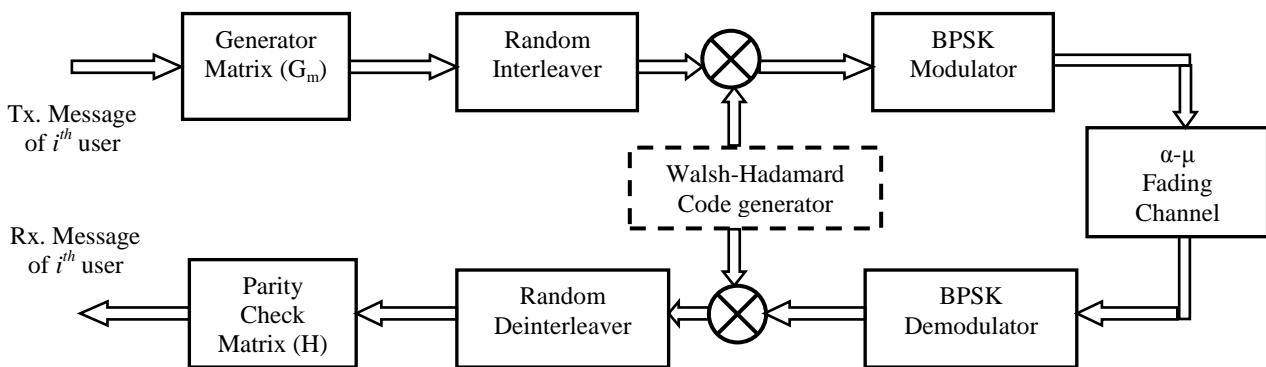


Fig. 2. Multi-user CDMA System model considered for simulation

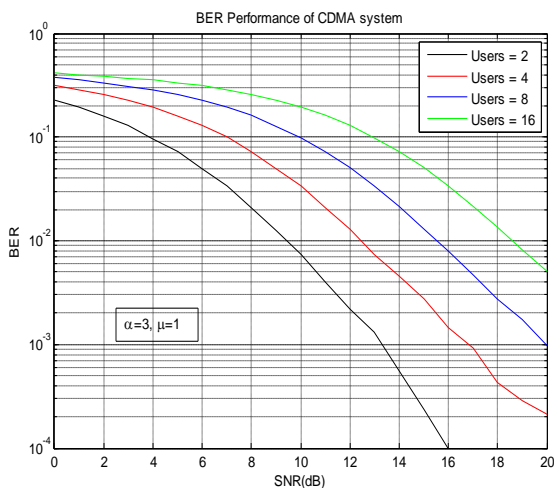


Fig. 3. BER of CDMA system over α - μ fading for $\alpha=3, \mu=1$

In Fig.3 the BER performance for $\alpha=3, \mu=1$ is shown. It is seen that BER performance for a given SNR decreases as the number users increases. At 10 dB SNR, the BER for 2, 4, 8 and 16 users is 7.0×10^{-3} , 3.4×10^{-2} , 9.5×10^{-2} , and 2.0×10^{-1} respectively.

In Fig.4 the BER performance for $\alpha=3, \mu=2$ is plotted. Here, it is noticed, at 10 dB SNR, the BER for 2, 4, 8 and 16 users is 1.5×10^{-4} , 3.2×10^{-3} , 3.0×10^{-2} , and 1.0×10^{-1} respectively.

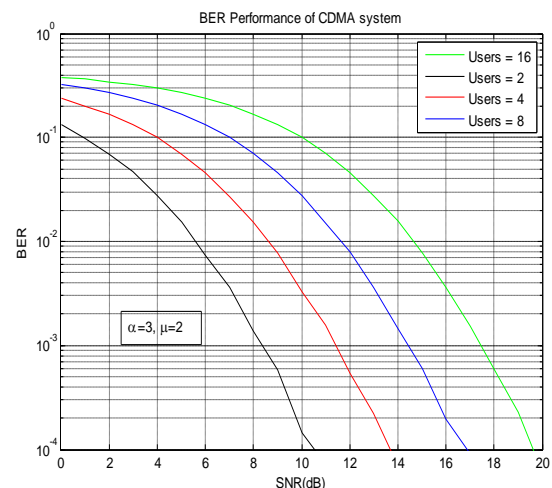


Fig. 4. BER of CDMA system over α - μ fading for $\alpha=3, \mu=2$

The plot of Fig.5 for $\alpha=3, \mu=3$ showing the simulation result for BER performance, wherein it is found that at 10 dB SNR, the BER for 4, 8 and 16 users is 4.5×10^{-4} , 9.5×10^{-3} , and 5.5×10^{-2} respectively.

In Fig.6, at 10 dB SNR, the BER for 4, 8 and 16 users is 4.0×10^{-4} , 1.2×10^{-2} , and 9.5×10^{-2} respectively.

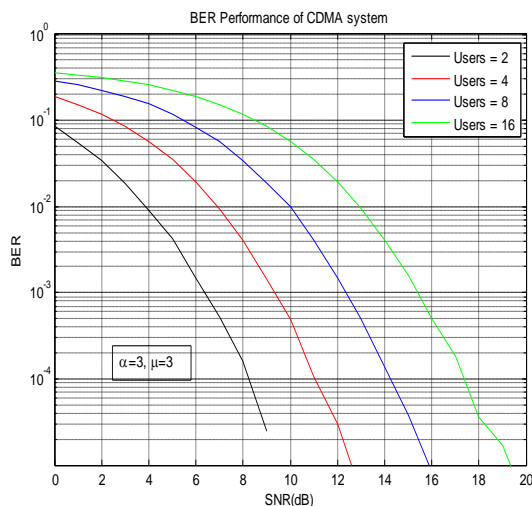


Fig. 5. BER of CDMA system over α - μ fading for $\alpha=3, \mu=3$

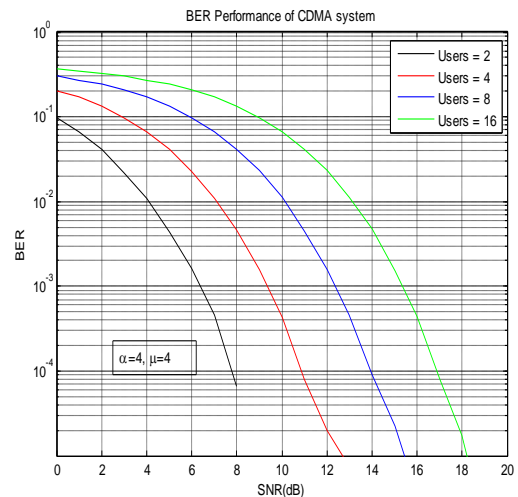


Fig. 6. BER of CDMA system over α - μ fading for $\alpha=4, \mu=4$

From the simulation results of Fig.3 to Fig.6, we find that at higher value of α and μ , the BER performance has significantly improved, due to high power α and large number of multipath clusters μ . It is also noticed that for larger number of users the BER performance decreases due to increase in multi user interference (MUI).

VII. CONCLUSION

In this paper α - μ fading channel has been briefly introduced. BER performance of single carrier multi-user CDMA system, with random interleaving are analysed. The effect of α and μ parameters on BER performance is brought out. It has been analyzed that increase in number of users adversely affects the error performance, due to increase in multi user interference. This analysis will be helpful for further investigation of link performance for multi-carrier CDMA system over α - μ channel.

ACKNOWLEDGMENT

This work is supported by Defence Research & Development Organisation (DRDO), Ministry of Defence, Government of India. The support is gratefully acknowledged.

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